

Analysis on Homogeneous Spaces and
Representation Theory of Lie Groups,
Okayama-Kyoto

**ADVANCED STUDIES
IN PURE MATHEMATICS 26**

Chief Editor: Eiichi Bannai (Kyushu University)

**Analysis on Homogeneous Spaces and
Representation Theory of Lie Groups,
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Preface

The representation theory of Lie groups and Lie algebras plays an important role in much of mathematics and mathematical physics in both classical and recent developments. Taking homogeneous spaces and representation theory as key words, the present editors organized the RIMS Project Research '97 (Chair: Toshio Oshima) during April 1997 to March 1998, hoping that representation theory are discussed from broader viewpoints than usual and seeking for new interactions of ideas and techniques with other branches of mathematics.

This volume is an outgrowth of the following activities of the RIMS Project Research '97:

- i) Hayashibara Forum on “Representation Theory and Homogeneous Spaces”, held at the Fujisaki Institute, Okayama, during July 27 - August 1, 1997,
- ii) the RIMS symposium “Analysis on Homogeneous Spaces and Representations of Lie Groups”, held at the Yukawa Institute for Theoretical Physics, Kyoto University, during August 4-8,
- iii) the RIMS workshop during August 11-15.

These symposia offered individual lectures on specialized topics, and the third one also offered some expository courses on current research. Accordingly, this volume contains both original articles and some expository surveys. The diversity of subjects of the contributions reflects currently very active areas in the representation theory of Lie groups, as well as various interactions with geometry of homogeneous spaces, automorphic forms, quantum groups, special functions, discrete groups, differential equations, and so on. Bringing together some of most active areas of the field today, we hope that this volume will serve not merely as proceedings but also as an excellent up-to-date guide to the representation theory of Lie groups.

The symposia were supported by the Hayashibara Foundation, the Japan Association for Mathematical Sciences, and the Grant-in-Aid by the Ministry of Education, Science, Sports and Culture, which enabled us to have participants from overseas. We are grateful to these institutions for generous financial support.

The preparation of the symposia was done mainly by the present editors with the support of Mr. Kazuya Masaki of Secretary General and Ms. Akiko Mizukawa, the Hayashibara Foundation, and Ms. Kazuko Suenaga, secretary of the RIMS.

We thank all the invited speakers of the symposia for their excellent

talks and contributed papers. We also thank all the participants for their active participation.

All the articles have been refereed. We would like to express our sincere gratitude to all the referees for their invaluable help to this volume.

Special thanks go to Ms. Kazuko Suenaga for her excellent job throughout the RIMS project '97 and the preparation of this volume.

Toshiyuki Kobayashi
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*All papers in this volume have been refereed and are in final form.
No version of any of them will be submitted for publication elsewhere.*

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