

Arrangements – Tokyo 1998

**ADVANCED STUDIES
IN PURE MATHEMATICS 27**

Chief Editor: Eiichi Bannai (Kyushu University)

Arrangements – Tokyo 1998

Edited by

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Advanced Studies in Pure Mathematics 27

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Preface

This volume of *Advanced Studies in Pure Mathematics* comprises the proceedings of a conference on hyperplane arrangements held at Tokyo Metropolitan University, July 13-18, 1998, as a Regional Workshop (RW) of the Mathematical Society of Japan. This “Workshop on Mathematics Related with Arrangements of Hyperplanes” celebrated the sixtieth birthyear of Peter Orlik, and featured daily lectures by Professor Orlik on “Arrangements and Hypergeometric Integrals.” These were followed by invited lectures, twenty-two in all, from an impressive group of mathematicians ranging across a wide range of topics from singularity theory, topology, combinatorics, and geometry, all related to hyperplane arrangements. Sixteen of these speakers have articles in these proceedings. The full schedule of talks from the workshop appear on succeeding pages.

Professor Orlik’s lecture notes will be published as MSJ Memoirs under the title *Arrangements and Hypergeometric Integrals*, co-authored by Peter Orlik and Hiroaki Terao.

There were eighty participants in the conference in total among whom twenty-five were overseas participants: USA (12), France (9), Korea (1), Mexico (1), Romania (1), and Russia (1).

The idea of organizing a meeting in honor of the sixtieth birthday of Peter Orlik first arose in conversations between Richard Randell and Michael Falk, two of his former students, in early 1995. These two served as the program committee for the meeting. This idea, together with the idea of having a Franco-Japanese seminar on singularity theory in Sapporo, was discussed and supported in an informal gathering, in Kyoto in 1996, of Jean-Paul Brasselet, Mutsuo Oka, Kyoji Saito, Tatsuo Suwa and Hiroaki Terao. Mutsuo Oka and Hiroaki Terao, as the organizers of the former conference (the “*Orlikfest*”), arranged for the meeting to be held at TMU, and obtained financial support from MSJ.

Peter Orlik’s work with Louis Solomon, building on work of V.I. Arnol’d and E. Brieskorn, really gave birth to the topological study of general complex hyperplane arrangements in the late 1970’s and early 1980’s. In the intervening years he has done much to nurture the development of the subject, with fundamental contributions to the ever-broadening research in the area, and with his two books, *Introduction to Arrangements* and *Arrangements of Hyperplanes*, with H. Terao, which are the first books to be published in this area. Orlik has also done much to encourage younger researchers in the field, including the two editors. The Tokyo conference, taking place ten years after the first conference

devoted to arrangements, the CBMS Regional Research Conference in Flagstaff, June 6-10, 1988, provided an opportunity to celebrate the growth and burgeoning vitality of the subject.

Orlik came to his work in arrangements from the field of singularity theory, and indeed, prior to 1988, research on arrangements of hyperplanes was customarily presented at singularities meetings. There is still a strong connection between the two subjects, some of which is evident within these pages. The 1998 Tokyo workshop took place one week after the international symposium “Singularities in Geometry and Topology” (July 6-10, 1998, Sapporo) which was an expansion of the Franco-Japanese seminar supported by CNRS (France) and JSPS (Japan), and organized by Jean-Paul Brasselet and Tatsuo Suwa. Many of the participants in the Tokyo workshop attended the Sapporo meeting as well, and several gave talks at both conferences. The original plan was to publish proceedings of the two conferences in one volume, edited by Brasselet, Falk, Suwa, and Terao. This turned out to be problematical because of its prospective size, and the planned volume was split into two. The proceedings of the Sapporo singularities conference is to be published as *Advanced Studies in Pure Mathematics, Singularities in Sapporo, 1998*, edited by J.-P. Brasselet and T. Suwa. We refer the reader to that volume for other articles bearing some relation to arrangements of hyperplanes. The four editors collaborated through much of the editing process of the proceedings, and the two of us express our deepest gratitude to Jean-Paul Brasselet and Tatsuo Suwa for their help in bringing the present volume to completion.

We would like to thank the Mathematical Society of Japan for their financial support for the workshop. Especially, we appreciate the financial support provided by Professor Atsushi Yoshikawa. We would also like to thank the editors of the *Advanced Studies in Pure Mathematics* series. Professor Eiichi Bannai, as the Chief Editor, gave us valuable advice and Professor Seiki Nishikawa helped us financially to prepare this volume. Finally, we would like to thank the participants of the meeting as well as mathematicians who served as referees of the papers for their cooperation.

Michael Falk and Hiroaki Terao

All papers in this volume have been refereed and are in final form. No version of any of them will be submitted for publication elsewhere.

Workshop on Mathematics Related to Arrangements of Hyperplanes

Tokyo Metropolitan University, July 13-18, 1998
(at TMU International House)

In honor of the 60th birthyear of Peter Orlik

July 13 (Mon)

9:40-10:30 **Peter Orlik** (Wisconsin) *Arrangements and hypergeometric integrals (1)*

10:45-11:45 **Kyoji Saito** (RIMS) *The polyhedron dual to the Coxeter arrangements*

13:30-14:30 **Luis Paris** (Bourgogne) *Intersection subgroups of complex hyperplane arrangements*

14:45-15:45 **Eriko Hironaka** (Florida State) *Boundary manifolds of line arrangements*

16:00-17:00 **Daniel C. Cohen** (Louisiana State) *Cohomology of fiber-type arrangements and Orlik-Solomon algebras*

July 14 (Tue)

9:40-10:30 **Peter Orlik** (Wisconsin) *Arrangements and hypergeometric integrals (2)*

10:45-11:45 **Hiroaki Terao** (Tokyo Metropolitan) *Antiinvariant differential forms and logarithmic forms of Coxeter arrangements* (with Louis Solomon)

13:30-14:30 **Bernard Teissier** (Ecole Normale Sup.) *Multiscale curvature concentration in Milnor fibers*

14:45-15:45 **Masaaki Yoshida** (Kyushu) *Intersection numbers of twisted (co)cycles*

16:00-17:00 **Christos Athanasiadis** (U. Penn.) *Deformations of Coxeter arrangements and their characteristic polynomials*

19:15-20:00 **Stefan Papadima** (Romanian Acad. Math. Inst.) *Milnor fibers of arrangements*

20:15-21:00 **Herbert Kanarek** (UNAM) *Gauss-Manin connections of arrangements*

July 15 (Wed)

9:40-10:30 **Peter Orlik** (Wisconsin) *Arrangements and hypergeometric integrals (3)*

10:45-11:45 **Richard Stanley** (MIT) *The distance enumerator of a hyperplane arrangement*

July 16 (Thu)

9:40-10:30 **Peter Orlik** (Wisconsin) *Arrangements and hypergeometric integrals (4)*

10:45-11:45 **Kazuhiko Aomoto** (Nagoya) *Picard-Lefschetz transformation for quasi-hypergeometric functions*

13:30-14:30 **Alex Suciu** (Northeastern) *Combinatorics and topology of complements of planes in \mathbf{R}^4*

14:45-15:45 **Michael Falk** (Northern Arizona) *Parallel connections, bundles and invariants of arrangements*

16:00-17:00 **Bruce Sagan** (Michigan State) *The k -consecutive arrangements*

July 17 (Fri)

9:40-10:30 **Peter Orlik** (Wisconsin) *Arrangements and hypergeometric integrals (5)*

10:45-11:45 **Toshitake Kohno** (Tokyo) *Elliptic KZ system, braid group of the torus and Vassiliev invariants*

13:30-14:30 **James Damon** (North Carolina) *The role of free divisors in the topology and combinatorics of nonlinear arrangements*

14:45-15:45 **Katsunori Iwasaki** (Kyushu) *Polytopes, invariants, and harmonic functions*

16:00-17:00 **Michel Deza** (Ecole Normale Sup.) *Embedding of all regular tilings and star-honeycombs*

July 18 (Sat)

9:40-10:30 **Peter Orlik** (Wisconsin) *Arrangements and hypergeometric integrals (6)*

10:45-11:45 **Sergey Yuzvinsky** (Oregon) *Cohomology of the Orlik-Solomon algebras and local systems (joint with Anatoly Libgober)*

13:30-14:30 **Mutsuo Oka** (Tokyo Metropolitan) *Geometry of cuspidal sextics and a new Zariski pair*

14:45-15:45 **Richard Randell** (Iowa) *Arrangements and the knot configuration space*

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