Combinatorial Methods in Representation Theory

ADVANCED STUDIES IN PURE MATHEMATICS 28

Chief Editor: Eiichi Bannai (Kyushu University)

Combinatorial Methods in Representation Theory

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Published for the Mathematical Society of Japan by

KINOKUNIYA COMPANY LTD. TOKYO, JAPAN This book was typeset by $\mathcal{A}_{\mathcal{M}}S$ - $T_{E}X$ and $\mathcal{A}_{\mathcal{M}}S$ - $I_{E}^{A}T_{E}X$, the $T_{E}X$ macro systems of the American Mathematical Society, together with the style files aspm.sty and aspmfm.sty for $\mathcal{A}_{\mathcal{M}}S$ - $T_{E}X$ written by Dr. Chiaki Tsukamoto and aspmproc.sty for $\mathcal{A}_{\mathcal{M}}S$ - $I_{E}X$ written by Dr. Akihiro Munemasa.

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Edited by the Mathematical Society of Japan.

Published for the Mathematical Society of Japan and distributed in Japan by Kinokuniya Company Ltd., Tokyo, Japan.

Distributed outside Japan by the American Mathematical Society.

ISBN 4-314-10141-5

2000 Mathematics Subject Classification. Primary 05xx, 17Bxx, 20Cxx; Secondary 05A15, 05A17, 05A19, 15A15, 16S30, 16S35, 16S99, 20F55, 33E17, 82B23.

Advanced Studies in Pure Mathematics 28 Chief Editor

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PRINTED IN JAPAN by Tokyo Shoseki Printing Co., Ltd.

Preface

This volume is a collection of 19 research and survey papers written by the speakers of the two international conferences:

1. "Combinatorial Methods in Representation Theory" (from July 21 to July 31, 1998), and

2. "Interaction of Combinatorics and Representation Theory" (from October 26 to November 6, 1998).

These conferences were held at RIMS (Research Institute for Mathematical Sciences, Kyoto University), as part of the Research Project of RIMS on "Combinatorial Methods in Representation Theory" for the academic year 1998–99. We had two visiting professors (John Stembridge from University of Michigan and Bernard Leclerc from Université de Caen) and 25 invited speakers from abroad during that period.

This field, which is developed by the interactions of Combinatorics and Representation Theory, is old and new. From the origin of the representation theory (A. Young, I. Schur, H. Weyl etc.), the study of the representations of specific groups and algebras inevitably incurred the combinatorial problems to be conquered. From the viewpoints of combinatorics, the combinatorial objects or structure occurring in this manner have interesting properties and beautiful symmetries in their own right. Since representation theory has strong wings, such as cohomology theory, harmonic analysis and algebraic geometry, it can fly up to the higher sky without any help of combinatorics. But if we stick to the down-to-earth calculations, which we believe to be essential in mathematics, we need to study their combinatorial aspects.

This volume contains such kinds of interesting papers and surveys stemming from representation theory, namely they treat representations of (affine) Hecke algebras and affine Lie algebras, combinatorial properties of Kazhdan-Lusztig polynomials, crystals and Gelfand-Zetlin bases for Lie (super) algebras, etc.

We thank all the participants to the above project for showing their interests during the program and thank the Mathematical Society of Japan for publishing this volume in the *Advanced Studies in Pure Mathematics* series. We also address our thanks to the Grant-in-Aid for Scientific Research and the Japan Association for Mathematical Sciences for their financial support.

Last but not least, we would like to express our hearty gratitude to Ms. Kazuko Suenaga and other secretaries in RIMS for their devoted assistance in the project.

Editorial Committee Masaki Kashiwara Kazuhiko Koike Soichi Okada Itaru Terada Hiro-Fumi Yamada

All papers in this volume have been refereed and are in final form. No version of any of them will be submitted for publication elsewhere.

CONTENTS

Susumu Arıkı — Robinson-Schensted Correspondence and Left Cells	1
Georgia Benkart and Seok-Jin Kang — Crystal Bases for Quantum Superalgebras	21
Francesco BRENTI — Approximation Results for Kazhdan-Lusztig Polynomials	55
Mihai CIUCU and Christian KRATTENTHALER — Plane Partitions II: ${\bf 5\frac{1}{2}}$ Symmetry Classes	81
Akihiko GYOJA, Kyo NISHIYAMA and Kenji TANIGUCHI — Invari- ants for Representations of Weyl Groups, Two-sided Cells, and Modular Representations of Iwahori-Hecke Algebras	103
Goro HATAYAMA, Yoshiyuki KOGA, Atsuo KUNIBA, Masato OKADO and Taichiro TAKAGI — Finite Crystals and Paths	113
Masao Ishikawa and Masato Wakayama — Minor Summation Formulas of Pfaffians, Survey and a New Identity	133
Alexander KIRILLOV, JR. and Alain LASCOUX — Factorization of Kazhdan–Lusztig Elements for Grassmanians	143
Bernard LECLERC and Jean-Yves THIBON — Littlewood-Richardson Coefficients and Kazhdan-Lusztig Polynomials	155
Alexander I. MOLEV — A Weight Basis for Representations of Even Orthogonal Lie Algebras	221
Tatsuhiro NAKAJIMA and Hiro-Fumi YAMADA — Schur's Q -functions and Twisted Affine Lie Algebras	s 241
Maxim NAZAROV — Capelli Elements in the Classical Universal Enveloping Algebras	261
Yuval ROICHMAN — On Permutation Statistics and Hecke Algebra Characters	287
Anne SCHILLING and Mark SHIMOZONO — Bosonic Formula for Level-restricted Paths	305
Toshiaki Shoji — Length Functions for $G(r, p, n)$	327

Takeshi Suzuki — Representations of Degenerate Affine Hecke Algebra and \mathfrak{gl}_n	343
Hiroyuki TAGAWA — A Recursion Formula of the Weighted Parabolic Kazhdan-Lusztig Polynomials	e 373
Yasuhiko YAMADA — Special Polynomials and Generalized Painlevé Equations	391
Manabu YAMAGUCHI — A Duality of a Twisted Group Algebra of the Hyperoctahedral Group and the Queer Lie Superalgebra	401