Taniguchi Conference on Mathematics Nara '98

# ADVANCED STUDIES IN PURE MATHEMATICS 31

Chief Editor: Eiichi Bannai (Kyushu University)

# Taniguchi Conference on Mathematics Nara '98

Edited by

Masaki Maruyama (Kyoto University) Toshikazu Sunada (Tohoku University)

Mathematical Society of Japan

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# Advanced Studies in Pure Mathematics 31 Chief Editor

#### Eiichi Bannai (Kyushu University)

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#### Preface

This volume is a collection of the research manuscripts written by the invited speakers of the Taniguchi Conference on Mathematics, Nara '98, which was held on December 15–20, 1998 at the Nara Prefectural Cultural Hall, Nara, Japan. The invited speakers were Hitoshi Arai, Spencer J. Bloch, Richard E. Borcherds, Kenji Fukaya, Mikio Furuta, Yoshikazu Giga, Masaki Kashiwara, Kazuya Kato, Janos Kollar, Maxim Kontsevich, Shigeo Kusuoka, Pierre-Louis Lions, George Lusztig, Paul G. Malliavin, Shinichi Mochizuki, Shigefumi Mori, Adrian Ocneanu, Kyoji Saito, Yum-Tong Siu and Srinivasa S. R. Varadhan.

The members of the organizing committe were Akira Fujiki, Mitsuru Ikawa, Kiyoshi Ito, Michio Jimbo, Shigeo Kusuoka, Masaki Maruyama, Shingo Murakami, Junjiro Noguchi, Kazuo Okamoto, Kyoji Saito, Toshikazu Sunada and Kenji Ueno.

For more than 20 years, Taniguchi Symposia of Mathematics had been held every year under the auspices of the Taniguchi Foundation chaired by Professor Kiyoshi Ito and the late Professor Shingo Murakami. The main objective of these symposia was to create opportunities for many mathematicians in Japan and foreign countries to exchange their ideas and research findings. More detailed accounts regarding the history, objectives and significant contributions of the Taniguchi Foundation is provided by Professor Shingo Murakami after this preface.

The set up of the Taniguchi Foundation ceased in 1999. In order to commemorate its many years of financial support to the mathematics activities, we decided to hold this final Conference which aimed to gather together people in mathematics who have ever attended (participated) in Taniguchi Symposia. The areas of subjects in this Conference were chosen to include all important and active fields of mathematics, and thus the topics in this proceedings are rather diversified. Nevertheless, these manuscripts are useful references for further research undertakings in their respective fields.

On behalf of the organizers, we wish to express our thanks to all invited speakers for kindly accepting our invitation and all participants for enthusiastically responding to the Conference. We would like to give our respects and to express our gratitude to the late Professor Shingo Murakami. For without his concerted effort and sustained support, it would be impossible to organize this Conference. Finally, we sincerely thank the Taniguchi Foundation for providing generous support to hold this Conference.

December, 2000 Toshikazu Sunada, Masaki Maruyama, Editor

All papers in this volume have been refereed and are in final form. No version of any of them will be submitted for publication elsewhere.

#### Contribution of Mr. Taniguchi to the Mathematical Community of Japan

#### Singo Murakami

Toyosaburo Taniguchi<sup>1</sup>, chairman emeritus of the board of Toyobo Corporation, established the Taniguchi Foundation in 1929 with his personal funds in accordance with his late father's will.

Mr. Taniguchi's aim has been to promote research in the basic sciences in Japan as well as to promote mutual understanding and friendship on an international level through international exchange of idea and research in the sciences.

Mr. Taniguchi attended the Third National Senior High School at Kyoto with the late Kiyoshi Oka and Professor Yasuo Akizuki, and their friendship continued through the years. In fact, while Oka had been doing research on the theory of the functions of several complex variables under the difficult conditions of World War II, Mr. Taniguchi supported him financially. In 1956, Professor Akizuki, then at Kyoto University, approached Mr. Taniguchi and informed him that the Mathematical Community in Japan was so short of funds that they could not afford to organize conferences or meetings to exchange their ideas. Mr. Taniguchi promptly instituted a division for mathematics within the Taniguchi Foundation. Thus, he became the sponsor of a summer seminar, headed by Professor Akizuki. The first seminar was held at Akakura in 1956 on "Manifold Theory from Differential Geometric Viewpoint" organized by Y. Matsushima and K. Nomizu, and attended by N. Iwahori, S. Nakano, I. Satake, S. Murakami and several others. Since that time the seminar has been held each year on various topics of mathematics. At times foreign scholars also participated in the seminar. Mr. Taniguchi supported the Japan-United States Symposium on Mathematical Education in 1964. In 1966, the Taniguchi Foundation sponsored the International Congress on Differential Equations at Katata to which Professor W. L. Baily, K. O. Friedrichs, K. Kodaira, J. J. Kohn, M. Kuranishi, R. S. Phillips, H. Rossi and D. C. Spencer were invited.

In 1974, Mr. Taniguchi contributed a huge amount of his personal funds to the then existing Taniguchi Foundation and began anew. At this time, Mr. Taniguchi promoted and sponsored an International Symposium on a small scale in various fields of sciences.

<sup>&</sup>lt;sup>1</sup>Mr. Toyosaburo Taniguchi passed away in 1994 at the age of 93.

In particular, Mr. Taniguchi was convinced of the value of limiting the number of participants in order to allow maximum flow of communication and personal contact. While it has been his desire primarily to raise the level of scientific thought and research in Japan, at the same time it was his dream to provide a forum where promising young scholars the world over may gather informally to exchange their thoughts, thereby contributing to mutual understandings, peace and the development of mankind in the world of the future. We respect this idea of Mr. Taniguchi, are grateful to him, and wish to cooperate with him in the future in his efforts to realize his goal.

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## Taniguchi Conference on Mathematics Nara '98

DATE: December 15–20, 1998

PLACE: the Nara Prefectural Cultural Hall, Nara, Japan

### PROGRAM:

## December 15 (Tuesday)

10:30 - 11:3	0 R. E. Borcherds
Ver	tex algebras
11:50 - 12:5	0 M. Kontsevich
Nor	-commutative algebraic manifolds
14:30 - 15:3	0 K. Fukaya
Lag	rangian submanifolds and mirror symmetry
18:00 - 20:0	0 <b>PL. Lions</b>
On	some mathematical problems in fluid mechanics

### December 16 (Wednesday)

10:00 – 11:00 <b>K. Saito</b>
Infinite dimensional Lie algebras and their $L$ -functions arising
from weight systems
11:30 – 12:30 M. Kashiwara
Micro-localization and contact transforms
14:30 – 15:30 <b>J. Kollar</b>
The topology of real and complex algebraic varieties
16:00 – 17:00 <b>S. Mochizuki</b>
The intrinsic Hodge theory of $p$ -adic hyperbolic curves

## December 17 (Thursday)

10:00 - 11:00	S. Mori
Ratio	nal curves on algebraic varieties – a survey and problems
11:30 - 12:30	G. Lusztig
Repre	esentation theory in characteristic $p$

## December 18 (Friday)

10:00 - 1	11:00	M. Furuta
	Spin st	ructure of 4-dimensional manifolds
11:30 - 1	12:30	K. Kato
	Bloch's	s conductor formula and $p$ -adic epsilon elements

- 14:30 15:30 **Y.-T. Siu**
- Applications of  $L^2$  estimates of  $\overline{\partial}$  to algebraic geometry 16:00 – 17:00 **P. G. Malliavin**

Construction of probability measure in infinite dimension

## December 19 (Saturday)

10:00 – 11:00 <b>A. Ocneanu</b>
Operator algebras, topology and quantum symmetry
11:30 – 12:30 <b>S. Kusuoka</b>
Approximation of expectation on diffusion model in
mathematical finance
14:30 – 15:30 S. J. Bloch
Algebraic cycles and differential forms
16:00 – 17:00 <b>H. Arai</b>
Harmonic analysis for singular elliptic differential operators
and its applications

## December 20 (Sunday)

10:00 - 11:00 Y. Giga Very fast diffusion and interface dynamics
11:30 - 12:30 S. R. S. Varadhan Large deviation and hydrodynamic scaling

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