

Groups and Combinatorics — in memory of Michio Suzuki

**ADVANCED STUDIES  
IN PURE MATHEMATICS 32**

Chief Editor: Eiichi Bannai (Kyushu University)

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**Groups and Combinatorics  
— in memory of Michio Suzuki**

Edited by

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## **Advanced Studies in Pure Mathematics 32**

**Chief Editor**

**Eiichi Bannai** (Kyushu University)

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## Preface

In honor of Professor Michio Suzuki's 70th birthday, a conference was held in July 1997, at the International Christian University, Mitaka, Tokyo. In early 1998, we published an informal proceedings of the conference, and then began the preparation of a book which would have celebrated the 70th birthday of Professor Suzuki. However, in February 1998, we received an email from Kazuko Suzuki (Professor Suzuki's daughter) informing us that her father had decided to return to Japan immediately, since he was suffering from liver cancer, and that his condition was extremely grave. We were greatly shocked by this news, because Professor Suzuki looked very well during his 70th birthday conference.

Professor Suzuki stayed in hospital in Tokyo. Some of us visited him regularly to discuss mathematics, as well as what we could do for him. Professor Suzuki knew that he would have only a few more months. Together with Professor Suzuki we discussed frankly the possibility that we must change the volume for his 70th birthday to a volume in his memory, and made detailed plans of the volume. Before Professor Suzuki returned to Japan, he had nearly completed a manuscript of about 150 pages, and he tried to give the finishing touches to this manuscript in hospital while fighting his illness. The priority was to make secure the publication of this manuscript, and with his approval, it was decided that the 150 page manuscript would be included in this volume (Professor Michio Suzuki passed away on May 31, 1998).

In the autumn of 1998, we sent invitations for contributions to this memorial volume to the invited speakers of the conference, as well as to researchers who had close ties with Professor Suzuki. Many of them willingly submitted their manuscripts to this volume.

We thought the most important task in this project was to properly handle the paper left by Professor Suzuki. Professor Helmut Bender was asked to check the overall correctness of the manuscript, and confirmation was received within a year (Professor Bender's opinion was that only minimum unavoidable corrections should be made to honor the writing style and the original flow of Professor Suzuki's thoughts). Based on Professor Bender's suggestions, and with the help of Kazuko Suzuki, some necessary corrections were made. In addition, Dr. Naoki Chigira was asked to check the mathematical details of the paper, since his recent interests had been close to the contents of the manuscript. Dr. Chigira took this job very seriously, pointed out many minor irregularities, gave many valuable comments, and also confirmed the overall

correctness of the paper. Except for very obvious misprints, the mathematical changes suggested by Dr. Chigira were not incorporated into the text (following Professor Bender's opinion concerning the style of the paper). We are extremely grateful to Dr. Chigira for his laborious contribution in checking the details of the manuscript. Any reader who is interested in the details of his comments should consult Dr. Chigira directly.

Professor Michio Suzuki had a great influence in group theory over the last 50 years. We believe that his work in the 1950's ignited work on the classification of finite simple groups, and in the 1960's and 70's he led its development. The classification was completed in the early 1980's by Aschbacher, Gorenstein, Thompson and many others. Professor Suzuki's most recent contribution to group theory is in this volume.

Although Professor Suzuki was affiliated with the University of Illinois from 1952 until his death, he had a profound influence on the development of group theory (and subsequently of algebraic combinatorics) in Japan. Through him, word of recent developments in group theory often reached Japan, in particular through his regular summer visits. His recommendations gave many of us opportunities to visit abroad for research and to help us find jobs. We believe it is fair to say that much of the respect given to group theory in Japan as a whole derives from that given to Professor Michio Suzuki as one of the top mathematicians in the world (of course, many other Japanese group theorists deserve credit: nonetheless, the influence of Professor Suzuki was extraordinary). The article by Koichiro Harada in this volume gives an excellent account of this.

It is unfortunate that Professor Suzuki passed away before we could show him our immense gratitude for leading us. Although they are private memories, each of us cannot forget the wonderful times we had visiting the University of Illinois and his home in Champaign, Illinois.

While editing this volume, it has been suggested by many mathematicians that Professor Suzuki's works are exceptionally important, and that they are worthy of being published as a collected work. We firmly agree with this opinion. At the same time, we feel that we are not well qualified for this important task, and hope that other, more qualified people, will take part. We also hope that the publication of this memorial volume will trigger such a project. We are willing to help with such a project in any way that we can, and we hope that Koichiro Harada will be involved.

We would like to thank many of those who helped towards the publication of this volume. It is regrettable that we cannot cite everyone's name. Besides those mentioned in the informal proceedings, we would

like to thank Professor Tadao Oda (former Chief Editor of ASPM) for allowing this volume to be published in ASPM series, and Professor Seiki Nishikawa for providing financial support from his Grant-in-Aid (from the Japanese government) for editing this volume. Hiroshi Suzuki supervised the work of changing the manuscripts to fit the style file of ASPM, with the help of Kenichiro Tanabe and Mitsugu Hirasaka. Finally, we would like to thank Naoko Suzuki (Mrs. Michio Suzuki) and Kazuko Suzuki, who helped us in many ways during the editing of this volume. The typing, as well as the corrections for Professor Michio Suzuki's last paper were made by Kazuko Suzuki with the additional help of Naoko Suzuki.

It has been almost 3 years since the death of Professor Michio Suzuki. We apologize for the long time it has taken us to put this volume in a publishable form. Although time seems to pass quickly, we will not let the vivid memories of Professor Suzuki and his mathematics fade away. We hope that this volume will renew people's memories, and also provide an opportunity for new generations of mathematicians to acquaint themselves with Professor Michio Suzuki and his mathematics.

Eiichi Bannai,  
Hiroshi Suzuki,  
Hiroyoshi Yamaki, and  
Tomoyuki Yoshida

*All papers in this volume have been refereed and are in final form.  
No version of any of them will be submitted for publication elsewhere.*

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**The 14th Algebraic Combinatorics Symposium**  
IN HONOR OF PROFESSOR MICHIO SUZUKI'S 70TH BIRTHDAY  
International Christian University, Mitaka, Tokyo, JAPAN  
July 14 – 17, 1997

**PROGRAM**

**July 14 (Monday)**

- 9:00 – 9:50 Walter Feit (Yale University)  
Integral Representations of Weyl Groups and Related Topics
- 10:10 – 11:00 Michael Aschbacher (California Institute of Technology)  
Quasithin Groups
- 11:20 – 12:10 Paul J. Flavell (The University of Birmingham)  
Solubility Criteria for Finite Groups
- 13:40 – 14:30 Alexander A. Ivanov (Imperial College)  
A Cover of the 3-local Geometry of  $Co_1$
- 14:40 – 15:10 Miller Maley (Princeton University)  
Hall Polynomials for Classical Groups
- 15:40 – 16:30 Eiichi Bannai (Kyushu University)  
Codes over Finite Rings and Finite Abelian Groups (A Survey)
- 16:40 – 17:30 Hiroshi Suzuki (International Christian University)  
Problems of Distance-Regular Graphs and Related Topics (A Survey)

**July 15 (Tuesday)**

- 9:00 – 9:50 Toshiaki Shoji (Science University of Tokyo)  
Representations of Finite Chevalley Groups
- 10:10 – 11:00 Tomoyuki Yoshida (Hokkaido University)  
Exponential Formulas Related to Finite Groups and Toposes
- 11:20 – 12:10 Gernot Stroth (Martin-Luther-Universität Halle)  
Small Modules
- Room A (Parallel Session)
- 13:40 – 14:05 Yoko Usami (Ochanomizu University)  
Principal Blocks with Extra-Special Defect Groups of Order 27

14:05 – 14:30 Katsuhiko Uno (Osaka University)  
Vertex of Non-Periodic Modules in the Auslander-Reiten Quiver of  
Finite Groups

14:40 – 15:05 Shigeo Koshitani (Chiba University)  
Morita Equivalent Blocks of Finite Groups

15:05 – 15:30 Atsumi Watanabe (Kumamoto University)  
Perfect Isometries and Glauberman Correspondences

Room B (Parallel Session)

13:40 – 14:05 Jack Koolen (Kyushu University)  
Are There Finitely Many Distance-Regular Graphs with a Given  
Valency?

14:05 – 14:30 Akira Hiraki (Osaka Kyoiku University)  
An Improvement of a Diameter Bound by Ivanov (Joint work with  
Jack Koolen)

14:40 – 15:05 Tatsuro Ito (Kanazawa University)  
Nonthin  $T$ -Modules of Endpoint 1

15:05 – 15:30 Yasuo Teranishi (Nagoya University)  
The Number of Spanning Trees and Laplacian Eigenvalues of a  
Graph

Room A (Parallel Session)

16:05 – 16:30 Kazumasa Nomura (Tokyo Medical and Dental Univer-  
sity)

Some Formulas for Spin Models on Distance-Regular Graphs, I

16:40 – 17:05 Brian Curtin (Kyushu University)

Some Formulas for Spin Models on Distance-Regular Graphs, II

17:05 – 17:30 Mitsuhiro Sawano (Kyushu University)

The Classification of Four-Weight Spin Models with Size 5

Room B (Parallel Session)

16:05 – 16:30 Hiroki Sasaki (Ehime University)

The mod 2 Cohomology Algebras of Finite Groups with Wreathed  
Sylow 2-Subgroups

16:40 – 17:05 Hiroshi Kimura (Ehime University)

Construction of Hadamard Matrices Using Dihedral Groups

### July 16 (Wednesday)

9:00 – 9:50 Helmut Bender (Christian-Albrechts-Universität zu Kiel)

Generalized Fitting  $\pi$ -Subgroups of Finite Groups

10:10 – 11:00 Bernd Stellmacher (Christian-Albrechts-Universität zu Kiel)

$J$ -Components in Finite Groups

11:20 – 12:10 Don G. Higman (University of Michigan)

Association Schemes and Geometries

13:40 – 14:30 Ronald Solomon (The Ohio State University)

The Semisimple Method for the Classification of the Finite Simple Groups

14:40 – 15:10 Michael Aschbacher (California Institute of Technology)

Groups Disconnected at the Prime 2

15:40 – 16:30 Koichiro Harada (The Ohio State University)

Professor Michio Suzuki's Contribution to the Theory of Finite Groups

16:40 – 17:30 Michio Suzuki (The University of Illinois Urbana-Champaign)

On the Prime Graph of Finite Simple Groups — An Application of a Method of Feit–Thompson–Bender–Glauberman

*(Banquet, from 18:00 at ICU Cafeteria West Room)*

**July 17 (Thursday)**

9:00 – 9:50 John McKay (Concordia University)

Moonshine — from Dedekind and Fricke, Klein to the Present Day

10:10 – 11:00 Masahiko Miyamoto (Tsukuba University)

The Finite Group Theory on Vertex Operator Algebras

11:20 – 12:10 Franz Timmesfeld (Justus-Liebig-Universität Giessen)

Identifications of Lie-type Groups.

Room A (Parallel Session)

13:40 – 14:05 Paul-Hermann Zieschang (Kyushu University)

The Classification of the Buildings of Spherical Type in the Light of the Theory of Association Schemes

14:05 – 14:30 Satoshi Yoshiara (Osaka Kyoiku University)

Subgroup Complexes for Finite Simple Groups

14:40 – 15:05 Michio Ozeki (Yamagata University)

A Polynomial Approach to the Covering Radius Problem for Ternary Self-Dual Codes

Room B (Parallel Session)

13:40 – 14:05 Masaaki Kitazume (Chiba University)  
Binary Codes Vertex Operator Algebras and Finite Automorphism  
Groups

14:05 – 14:30 Hiromichi Yamada (Hitotsubashi University)  
Ternary Code Vertex Operator Algebras

14:40 – 15:05 Norio Yamazaki (Kyushu University)  
On Local Structures of Some Group Association Schemes

15:05 – 15:30 Masato Tomiyama (Osaka Kyoiku University)  
Characterization of the Group Association Scheme of  $PSL(2, 7)$

Room A (Parallel Session)

15:40 – 16:05 Masaaki Harada (Yamagata University)  
New 5-Desings Constructed from the Lifted Golay Code over  $Z_4$

16:05 – 16:30 Takeshi Kondo (Tokyo Women's Christian University)  
Some Examples of Non-Solvable Unramified Extensions over Quadra-  
tic Fields

Room B (Parallel Session)

15:40 – 16:05 Mitsugu Hirasaka (Kyushu University)  
On Association Schemes with a Nonsymmetric Relation of Valency  
4

16:05 – 16:30 Akihiro Munemasa (Kyushu University)  
Primitive Trinomials and Orthogonal Arrays over  $GF(2)$

Room A

16:40 – 17:30 Hiroyoshi Yamaki (Kumamoto University)  
Prime Graphs (Joint work with N. Chigira and N. Iiyori)